

# Interest Rates and Fundamental Fluctuations in Home Values

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## *Abstract*

Fluctuations in housing prices are relevant to wealth accumulation, labor mobility, consumption, macroeconomic volatility, and financial market stability. However, it is *ex ante* difficult to know when housing price movements are due to fundamentals, such as changes in the user cost of capital, versus irrational exuberance. I propose combining the canonical urban economics Alonso-Muth-Mills model and Poterba (1984, 1990) housing asset-pricing equation to form grounded theoretical expectations about the impact of changes in the user cost of capital on home values. I show rental prices and rental expenditures to be endogenous to interest rates, which limits the applicability of conventional price-to-rent ratios. Expected changes in home values can be expressed as simple functions of the supply elasticity of housing, and the initial share of land relative to prices in a city. The simple formula can be used to diagnose and underwrite home valuations under the null hypothesis of a common shock to the user cost of capital. Empirically, I find that housing supply elasticities and land shares as of 1990 predicted 50% of the variance in price growth during the past boom. Deviations from theoretical growth mean-reverted dramatically during the bust period.

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# 1 Introduction

Housing prices are essential to account for macroeconomic fluctuations. Consumption of housing and related expenses are major drivers of aggregate demand. And the weight of mortgage lending on the banking system renders the value of housing assets critical for financial market stability. How housing values are formed, sustained, and changed is also important to understand housing markets and policies per se, wealth accumulation and bequests, the portfolio decisions of families, regional mobility, access to credit and consumption, and many other relevant economic decisions and outcomes.

In practice, however, it is difficult to know whether housing price movements are due to economic fundamentals tied to predictable components in the long-term demand and supply of the asset, to changes in local credit conditions, or to irrational exuberance. The asset-pricing approach to housing values, as developed by [10], [11], [9] and subsequent research, has arguably not provided a comprehensive framework to understand and forecast local housing price fluctuations. While [10] formally incorporates housing supply, explicit formulae for assessing the importance of land supply, land shares, and construction costs were left unmodelled in the original contribution. This perhaps explains why many researchers have applied the asset-pricing framework by discounting a stream of rental prices, treating rents as completely exogenous to the asset-pricing equilibrium.

Recent empirical contributions have nevertheless acknowledged the importance of differences in local housing supply to mediate the impact of changes in the user cost of capital. These papers explicitly incorporate estimates of supply elasticities to account for differences in the impact of increased access to collateralized credit. However, this new research has largely ignored the economics of land value formation, thereby missing the crucial aspect of this market. Moreover, this research has not attempted to provide prescriptive and predictive associations between land supply and housing prices.

In contrast, research following the canonical Alonso-Muth-Mills (AMM) model has always focused on land use and its rental price.<sup>1</sup> The existence of Ricardian land rents has been shown critical to understand urban development patterns and growth. This literature has explicitly modelled the conversion of rental cash-flows into land values via discounting, and has rigorously considered the

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<sup>1</sup>This is a very extensive literature. examples of which are [13], [14],[15],[2].[1] provides a masterful exposition of the model and the literature.

physical construction costs of improvements on land and real structures. A large associated empirical body of work has studied the capitalization of environmental attributes that are supplied inelastically into land rents and values.

Nevertheless, most of the urban economics literature has not been geared to studying the impact of changes in the cost of capital on asset prices. This literature has largely focused on *within-city* results for stylized homogeneous monocentric cities, or the general equilibrium properties of city size distributions. This focus may have prevented a wider empirical applicability of the urban economics model to study time-series and panel housing asset-price variation across heterogeneous cities.

In this context, I propose combining the insights of the urban model and asset-pricing frameworks. I demonstrate that adding a theoretical structure about the nature of land and housing supply to the basic housing asset-pricing model leads to highly productive insights. Focusing on the expected relationship between housing prices and changes in the user cost of capital, I derive a simple asset-pricing equation for housing that is based on a parsimonious number of fundamental economic parameters. The equation should help forecast relative housing price trends by diagnosing out-of-equilibrium outcomes. Such analysis of non-fundamental fluctuations should assist in the underwriting process of real estate and related financial assets.

The conceptual framework herein takes very seriously the physical nature of housing assets and the basic replicability of housing structures at replacement cost ([6], [8]). It then complements these foundations with insights about the pricing of the whole asset taking into account land supply and demand elasticities, and the discounting of future Ricardian rents.

Combining insights from the two canonical models we can capture house values in a metropolitan area by the price of a home in a representative location. I then focus on simple, substantive economic parameters of interest in order to produce clear insights and to assist in the applicability of the model to valuation exercises.

The intuitions from the application of the AMM model to asset pricing are as follows: consider a reduction in the user cost of housing capital, perhaps driven by lower interest rates or efficiency improvements in the financial sector. Reductions in the cost of capital should imply automatic reductions in the leasing costs (notional rental prices) of newly-built housing structures. For instance, considering a manufactured mobile home, changes in lending rates change the monthly leasing costs to the user without *necessarily* affecting the costs of production. More generally, since the building industry is very competitive and labor-intensive [8], production prices of new housing structures

may not change much.

Because existing properties are competing in the market with newly-built homes, the lower implicit leasing costs of new housing structures in fringe suburban or exurban locations should translate into lower leasing costs of housing structures elsewhere in the city.

The response of land rents (notional or explicit) to changes in the leasing costs of structural capital is ambiguous. On the one hand, distant exurban locations with large housing structures and low land costs are now substantially cheaper and look relatively more attractive to consumers; in order to compensate dwellers of smaller homes, their relative land rents should decrease, and more so in locations where land is used very intensively. On the other hand, the lower leasing costs of housing structures imply increased demand and more construction. Since land -together with structures- is required to produce more homes, this should also increase the demand for land, thereby pushing up Ricardian land rents *in all locations*. Overall -in equilibrium- implicit rental prices or user costs for the land-plus-structure composite *should decrease for the representative consumer*. However, consumers in land-intensive cities with very inelastic land supply experience smaller reductions in the overall monthly cost of ownership.

The model establishes that rents in the long run are directly endogenous to the user cost of capital, *even before taking into account general equilibrium effects via changes in the number of available homes*. Asset valuation models based on discounting current or past rents using the [10] formula (or simpler capitalization-rate methods) will yield misleading conclusions;<sup>2</sup> furthermore, the degree of such errors is heterogeneous and contingent on initial land shares and the local elasticity of housing supply.

Rental payments for a home are not equivalent to rental prices: they take into account both the quantity of housing consumed by the representative dweller and the rental price per square feet. The response of total housing rental payments will therefore be a function of the elasticity of demand for space, the share of land on housing value, and the supply elasticity of land.

Finally, the overall impact on housing prices depends on i) a discounting effect (because of a lower denominator in the asset-pricing discount formula); and ii) changing land rents. The first factor depends *exclusively* on the initial share of land inasmuch as structural replacement costs do

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<sup>2</sup>Of course, the model in Poterba (1984) allows for the rental or service flow of housing capital to be a function of housing supply  $R(H(t))$ , and so takes into account the general equilibrium effects of changes in user costs via increases in the housing stock ( $H$ ) caused by changes in asset prices. However, the model here suggests that direct effects via the leasing costs of structural capital are critically important and do not necessarily require changes in housing capital stock:  $R(v, H(t))$ , where  $v$  is the user cost of capital.

not change. Conventional discounting or rent-to-price approaches are thus fundamentally flawed for the long-term valuation of real estate assets because *structural leasing costs* should decrease one-to-one with changes in user costs.

In addition, the second effect implies the endogeneity of *Ricardian land rents* to changes in interest rates. The direction of this effect is ambiguous theoretically. However, it is certain that *land* values should increase relatively more in cities with more inelastic housing supply.

Under the assumption that the elasticity of housing demand is constant across locations, we can therefore fully predict the theoretical *price* impact of changes in the user costs of housing capital across locations by using only information about initial land shares (available from [3]) and the elasticity of housing supply (available from [12]).

Using these insights, I estimate an empirical model of housing price growth by metropolitan area between 2000 and 2005. As the theory predicts, we can go a long way explaining the heterogeneity in the run-up in housing prices in this period by using land shares and housing supply elasticities as predictors. In fact, I estimate that about 50% of the variation in housing prices during the boom was truly based on fundamentals (a common shock to the cost of capital as mediated by the relevant local economic parameters). This theoretically-consistent portion of the run-up in housing prices did not mean-revert in the bust period (2005-2008). In metropolitan areas where the price run-up was justified by the theory, prices did not decline much, if at all, during the housing bust. This framework could also possibly account for some of the cross-country differences in the magnitude of housing price declines since 2005.

Conversely, growth in prices that was not predicted by the model mean-reverted dramatically in the bust period. In the areas where the theoretical model would have predicted lower housing growth than experienced, the model was finally proved accurate after the bust.

Note that, in this paper, I do not address the question of how non-fundamental fluctuations - the so-called housing bubble and burst - arose in specific local housing markets in the US and many other countries. At this point, there is no consistent and widely-accepted economic model for out-of-equilibrium outcomes and their timing of reversion to long-run fundamental equilibria. Generating a comprehensive theory of "bubbles" will require further research including economic and other behavioral perspectives.<sup>3</sup> However, the paper provides a theoretically-grounded formula to assess if relative housing prices in specific locales are likely to be deviating from fundamentals,

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<sup>3</sup>A preliminary step in that direction has been undertaken by [7] Note that this paper assumes the existence of a bubble and does not consider the existence of genuine theoretical impacts of changes in the user cost of capital.

and shows that prices eventually revert to theoretically-grounded relative valuations. The formula developed in the paper could be therefore useful to value and underwrite risks inherent to real estate lending and investments, perhaps assisting to reduce the incidence of short-run behavioral deviations from fundamentals.

The paper proceeds as follows. Section 2 introduces the model, focusing on the expected impact of changes in the cost of housing finance. Section 3 discusses the empirical evidence: the permanence of theoretically-grounded housing price inflation but strong mean-reversion of unexpected deviations from the theory during the past boom-bust cycle. Section 4 concludes.

## 2 Model

### 2.1 Setup

The most parsimonious AMM structure that fully captures the key issues is as follows: there are a number of cities where homogeneous consumers live. Consumption in each city equals ‘real’ wages in the city (the marginal productivity of labor plus amenities), minus housing rental payments, minus the (monetized) costs of commuting to the Central Business District (CBD), where all jobs are located. Each individual is also a worker and lives in a separate house, so that the number of housing units equals population. Each square foot of housing is produced with a square foot of land and one unit of housing structure. Note that this is a shorthand to model the strong complementarity between land and structural capital in the production of housing.<sup>4</sup> We can think of each consumer as renting the use of the land and leasing the housing physical structure from landlords in competitive markets. However, the consumer could be renting the land and leasing the housing capital from herself (ownership), in which case we can think of *rents* as the notional user cost of housing/land *excluding asset value appreciation*.

At any point in time, the preferences of the homogenous consumer in city  $k$  can be captured by the instantaneous utility function:  $U(C_k, d, h) = w_k - tD + \frac{h^{1-\mu'}}{1-\mu'} - rh$ , where  $w_k$  stands for the real wage in the city (inclusive of a package of local amenities),  $h$  for the units of land/housing-space consumption,  $r$  for the rent *per unit of housing-space consumption* (or monthly financial and maintenance costs of ownership per square-foot),  $t$  for a parameter scaling to the monetary cost per distance commuted, and  $D$  for the distance of the consumer’s residence to the CBD.

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<sup>4</sup>The main conclusions carry over by simply assuming some kind of land/structure complementarity, but the extra complexity would not add much to the model.

Maximizing utility with respect to  $h$  yields the following demand function:  $h = \left(\frac{1}{r}\right)^\mu$ . The constant demand elasticity of housing is the negative of  $\mu = \frac{1}{\mu'}$ .

As in conventional AMM models (Brueckner, 1987), a non-arbitrage condition defines the rent gradient with respect to distance: all city inhabitants attain utility  $\overline{U}_k$  via free mobility and competition in the land markets. Hence we must have that  $\frac{dr}{dD} = -\frac{t}{h} = -tr^\mu$ .<sup>5</sup> From this differential equation we can obtain the general functional form for rents as a function of distance to the CBD (the bid-rent function):  $r(D) = [(1 - \mu) [C_1 - tD]]^{\frac{1}{1-\mu}}$ , where  $C_1$  is a city-specific parameter.

Now -on the supply side- express rental prices in a notional identity as the sum of the leasing cost of the homogeneously built physical structure ( $r_s$ ) and the Ricardian rent of land ( $r_l(D)$ ):  $r(D) = r_s + r_l(D)$ ; I will henceforth refer to the two inputs in the production of a housing unit as *land* and *structures*. The rental expenditure for each household corresponds to  $R(D) = h(D) \cdot r(D) = h \cdot r_s + h \cdot r_l(D)$ .

Expected future land rents are a function of city-wide economic growth, as modelled later. Note that, for simplicity, *I will be using the notation developed throughout to describe the value of the variables as of the initial period  $\tau = 0$  whenever the time variable is not specified.* Also note that all variables are expressed in real terms (net of monetary inflation).

The owners of the land and structures operate under no-arbitrage conditions in competitive asset markets with risk-neutral arbitrageurs. Consider the time-invariant *gross* real user-cost rate to housing investments  $v$  (excluding capital appreciation). Following Poterba (1984),  $v = [\delta + k + (1 - \theta)i - \theta\pi]$ : landlord-investors in real estate have to pay for physical maintenance cost rates ( $\delta$ ), local real estate tax rates ( $k$ ), and the after tax cost of capital  $(1 - \theta)i$  (with  $i$  being the interest rate and  $\theta$  the marginal tax rate), but benefit from tax deductions on nominal inflation  $\pi$  imbedded in the interest-rate deduction. As in Poterba's (1984, 1990) market asset-pricing equation, because  $v \cdot p(\tau, d) = \overset{\circ}{p}(\tau, d) + r(\tau, d)$  at any point in time, it follows that  $\overset{\circ}{p}(\tau, d) = [\delta + k + (1 - \theta)i - \theta\mu] \cdot p(\tau, d) - r(\tau, d)$ .<sup>6</sup>

Existing owners, however, face rental competition from new development, which is assumed to happen at the city's boundary located at a distance  $\Phi$  from the center. This is a key aspect, because it ensures that the leasing cost of structures in the city's interior cannot deviate from that of new

<sup>5</sup>To see that, realize that  $\frac{d\overline{U}}{dD} = \frac{\delta U}{\delta d} + \frac{dU}{dr} \frac{\delta r}{\delta d}$  and apply the envelope theorem  $\frac{dU}{dr} = \frac{\delta U}{\delta r} + \frac{\delta U}{\delta h} \frac{\delta h}{\delta r}$  with  $\frac{\delta U}{\delta h} = 0$ .

<sup>6</sup>Once could define the *net* user cost of capital rate as  $[\delta + k + (1 - \theta)i - \theta\mu] - \pi_e$ , where  $\pi_e$  is the appreciation rate of the asset with:  $\pi_e \cdot p(\tau, d) = \overset{\circ}{p}(\tau, d)$ . This is a less convenient formulation in this model because  $\pi_e$  will be shown to change if land values grow faster than the inflation and quality-adjusted cost deflator in the building sector.

housing at the border. At  $\tau = 0$  competitive developers sell newly-built homes at cost: the land value plus economic costs of construction (which include all relevant wages, materials, returns to capital, and taxes). On the supply-side for new homes, therefore  $p(\Phi) = cc + p_l(\Phi)$ : the price of the asset per square feet at the border ( $p(\Phi)$ ) is equal to the construction cost ( $cc$ ) plus the price which the developers are paying for the land ( $p_l(\Phi)$ ). I assume that construction costs do not change *in real terms* in the long-run, an assumption which is supported by empirical evidence.<sup>7</sup>

On the other hand, since the market for the supply of rental services is also perfectly competitive, prices at the border have to equal the present discounted value of the leasing cost of the structure plus the present discounted value of land rents (which is captured by the price of land  $p_l(\Phi)$ ). The asset price at the border is therefore pinned down by the arbitrage condition:  $cc + p_l(\Phi) = \frac{r_s}{v} + p_l(\Phi)$ , from which we infer  $r_s = cc \cdot v$ .

At the border there is no alternative use for land and many owners who are willing to rent its use and therefore we will assume w.o.l.o.g. that land rents at the development boundary are zero:  $r_l(\Phi) = 0$ .<sup>8</sup> Hence we have that rents at the border are simply equal to the leasing cost per structure:  $r(\Phi) = [(1 - \mu) [C_1 - t\Phi]]^{\frac{1}{1-\mu}} = r_s = cc \cdot v$ , which implies that:

$$r(D) = \left[ 1 + \frac{(1 - \mu)t(\Phi - D)}{[v \cdot cc]^{1-\mu}} \right]^{\frac{1}{1-\mu}} v \cdot cc \quad (1)$$

The bid-rent schedule in the above equation implies that rents at location  $D$  can be therefore expressed as the annuitized cost of construction multiplied by a "Ricardian markup factor."

In steady state, individuals are indifferent between the city and non-metro exurban locations. Outside of the city, individuals pay only the leasing costs of structures (no land rents), do not commute, and obtain real wages that are equal to  $\underline{w}$  (inclusive of consumption of all taxes and amenities with public-good characteristics).

We can focus now on the individuals at the border of the city - living at distance  $\Phi$  - who must be indifferent between the city and exurban locations. Since rents at the border and outside of the city are equal to the leasing price of the structure  $cc \cdot v$ , it follows that  $h(\Phi) = \left(\frac{1}{vcc}\right)^\mu$ .

Hence, utility at the city border is  $V(\Phi) = w_k - t\Phi - \frac{[vcc]^{1-\mu}}{1-\mu}$ , which is pinned down by the utility outside of urban areas:  $V(Exurb) = \underline{w} - \frac{[vcc]^{1-\mu}}{1-\mu}$ . Therefore  $\Phi = \frac{w_k - \underline{w}}{t}$ . W.o.l.o.g we assume that real wages in the exurbs stay constant at  $\underline{w}$ , so that we can posit a known *differential* growth

<sup>7</sup>However, extensions with construction cost inflation or deflation above CPI are straightforward.

<sup>8</sup>Note that we could include the opportunity cost of land to non-residential uses as part of the cost of construction (another cost of conversion to residential use).



rate of real wages in the city  $w_k(\tau)$  which, in turn, yields the expected city geographic growth path  $\Phi(\tau)$ .

To translate geography into population levels, I assume a circular city where geographic or regulatory land constraints make construction unfeasible in some areas: only a sector (share)  $\Lambda_k$  of the circle is developable. The radius of each circle around the CBD is  $2\pi \cdot D \cdot \Lambda$  and the number of households in each is  $\frac{2\pi D \Lambda}{h(D)}$ . Population in the city at  $\tau = 0$  is therefore defined by the number of

households as delimited by the following equation:  $H_k = 2\pi \Lambda \int_0^{\Phi} x \cdot r(x)^\mu dx$ .

In order to shed light on first-order pricing dynamics, I consider a notional economy in steady-state and focus on the impact of unexpected changes on  $v$ . This is a good approach in housing markets, where city-wide rental growth is persistent, owners lock into long-term mortgage rates which already incorporate future expectations about interest rates and real estate-specific risk premia, the tax system is not expected to change often, and transactions are infrequent. We are thus working within a Jorgensonian environment with known future user costs of capital and reversible investments: option-pricing extensions that consider alternative random processes for city-wide productivity, interest rates, and the other components of user costs are straightforward to incorporate in extensions of the model, but do not add first-order insights to the analysis here at substantial notational and transparency cost.<sup>9</sup>

I focus on current prices and drop time-zero subscripts henceforth for simplicity. Integrating, using the evolution of rents per square foot, we obtain (derivation 1, all derivations and proofs in appendix)

$$p(\tau = 0, D) = \int_0^{\infty} r(\tau, D) \cdot e^{-v\tau} \cdot d\tau = \int_0^{\infty} \left[ 1 + \frac{(1 - \mu)t(\Phi(\tau) - D)}{[v \cdot cc]^{1-\mu}} \right]^{\frac{1}{1-\mu}} v \cdot cc \cdot e^{-v\tau} \cdot d\tau$$

Note that -in this urban economics perspective of housing asset pricing - one does not obtain a homogeneous discount factor across locales within a city, in contrast with Poterba (1984, 1990). Note further that housing rents -*conditional on a level of housing supply*- are now also a *direct* function of the user cost of capital via the competition of rental units in the city with the leasing

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<sup>9</sup>In a number of conventional models, introducing a random process in the expectations about future interest rates leads to alternative discount factors in the asset pricing equation (Cochrane, 2001). Define this discount factor to be  $\kappa$ . To assess the impact of changes in interest rates on prices ( $\frac{dp}{dv}$ ) we can add considerations about the impact of changes in interest rates on the discount factor:  $\frac{dp}{dv} = \frac{\delta p}{\delta \kappa} \cdot \frac{\delta \kappa}{\delta v}$ . The expression  $\frac{\delta \kappa}{\delta v}$  is not the focus in this paper, but the model can accommodate extensions in this direction.

costs of structures in exurban locations. In conventional asset pricing models this relationship appears only indirectly: lower costs of capital justify higher prices and more construction, which pushes up the supply of housing thereby reducing rents eventually.

While the relative growth rate of rents will differ across locations within a city, it will be useful to define a representative location  $\tilde{D}$  in which housing rents are growing at a fixed growth rate  $g$ . At  $\tilde{D}$ ,  $r(\tau, \tilde{D}) = \left[1 + \frac{(1-\mu)t(\Phi-D)}{[v \cdot cc]^{1-\mu}}\right]^{\frac{1}{1-\mu}} v \cdot cc \cdot e^{g\tau}$ . While this can be interpreted as an approximation for the median-valued home or any other relevant data moment, one can easily show that there exists a city productivity path  $\widetilde{w_k(\tau)}$  that can rationalize exponential rental growth for any distance  $\tilde{D}$  (derivation 1). In the representative location we have that:

$$p(\tau = 0, \tilde{D}) = \left(\frac{v}{v-g}\right) \left[1 + \frac{(1-\mu)t(\Phi - \tilde{D})}{[v \cdot cc]^{1-\mu}}\right]^{\frac{1}{1-\mu}} cc$$

Or simply  $p(\tilde{D}) = \frac{r(\tilde{D})}{v-g}$ . Housing prices are equal to construction costs times the Ricardian markup, multiplied by a city-growth premium on such markup  $\left(\frac{v}{v-g}\right)$ . While housing values outside of  $\tilde{D}$  take on a hypergeometric formula, the intuitions contained in the above equation carry forward to such locations.

## 2.2 The impact of changes in the user cost of housing capital

In this section I study the impact of changes in housing rents and prices derived from a change in the user costs of capital. I assume that arbitrarily high moving costs do not allow people in the city to leave in response to changes in the cost of capital, a good assumption for the impact of such changes in the short to medium run. We have not seen *major* population reallocations following the recent past low interest rate environment around the globe, which makes this an appropriate assumption. Note that there are four main effects of exogenous reductions in the user cost of capital  $v$ :

1. A decrease in structural leasing costs.
2. A corresponding increase in the demand for space: we should see new construction and larger homes (also via additions and renovations of existing houses) and counterbalancing increases in land rents everywhere in the city. This effect *shifts-out* the land bid-rent schedule.
3. We should also see a counterbalancing decrease in relative land rents *in locations that are land-intensive*. Absent this effect, such locations would now be less competitive vis-a-vis structure-intensive areas. This effect *flattens out* the land bid-rent schedule.

4. A direct discounting effect in the price equation that is applicable only to land (structural values are pinned down by replacement costs).

In order to find the change in rental prices we will first see how much utility has changed for an individual living always at the border- at a distance that may be changing endogenously with the user cost of capital. Hence we will be able infer changes in utilities and rents in the rest of the city at each fixed distance. Recall the indirect utility of individuals living at the development fringe:  $V(\Phi) = w_k - t\Phi - \frac{[vcc]^{1-\mu}}{1-\mu}$ . Differentiating with respect to  $v$ , and using the envelope theorem:  $\frac{dV(\Phi)}{dv} = -t\frac{d\Phi}{dv} - \frac{dr(\Phi)}{dv}h(\Phi)$ .

Note that, since the demand for housing is encouraged by the lower leasing costs of structures, new construction is likely to happen and the city will be expanding. Recall that at the border land rents will always be zero:  $r(\Phi) = v \cdot cc$ , and thus  $\frac{dr(\Phi)}{dv} = cc$ . Therefore:  $\frac{dU}{dv} = -t\frac{d\Phi}{dv} - cc \cdot h(\Phi)$ . Because everyone in the city must be indifferent, the same change in utility applies to someone who stays at location  $D$ . Thus, taking into account the envelope theorem again:  $\frac{dV(D)}{dv} = -\frac{dr(D)}{dv} \cdot h(D) = -t\frac{d\Phi}{dv} - cc \cdot h(\Phi)$ . Rearranging we obtain  $\frac{dr(D)}{dv} = cc \cdot \frac{h(\Phi)}{h(D)} + \frac{t}{h(D)}\frac{d\Phi}{dv}$ . Now we can prove the following relationship describing the *elasticity* of rents at location  $D$  with respect to changes in the user cost of capital:

*Proposition 1*

$$\frac{dr(D)}{dv} \frac{v}{r(D)} = \sigma_r^S \cdot \frac{h(\Phi)}{h(D)} \left(1 - \frac{\mu}{\varepsilon^S}\right)$$

Where  $\varepsilon^S$  is the city-wide elasticity of housing supply with respect to rents, we recall  $\mu$  as the elasticity of demand for space (sign reversed), and  $\sigma_r^S$  is the share attributable to the leasing costs of the structure on total rents. Rental prices for housing are directly endogenous to discount rates: decreases in the user cost of housing structures should be associated with decreases in notional rental prices. This effect is straightforward for the portion of rental prices deriving from structural leasing costs ( $\sigma_r^S$  in the expression above): Given fixed physical costs of production, leasing the structure should be cheaper with lower interest rates. Since existing homes are competing with newly-built structures for owners/renters -in equilibrium- this effect *should* apply to all housing structures in the city (*Graphs A and B*).

Less intuitively, this process is reinforced by *partial* decreases in land rents in some locations. Consider the absolute change in rents at any point  $\frac{dr(D)}{dv} = cc \frac{h(\Phi)}{h(D)} \left(1 - \frac{\mu}{\varepsilon^S}\right)$ . Note that the average home is smaller than newly-constructed structures in the city's outlying areas. At the city's border, larger structural costs relative to land values render leasing a home much cheaper whenever the

user cost of capital is low. We therefore have to compensate dwellers in high land-rent locations via rental reductions at the ratio of  $\frac{h(\Phi)}{h}$  (*Graph D*). Nevertheless, while both structural leasing costs and land rents are decreasing in locations where land is valuable, it is still true that in locations with high land values the combined effects may be relatively smaller *as a percentage of initial rents*.

On the other hand, the growth in the demand for land in the city somewhat counterbalances the previous effects (*Graph C*), as captured by  $(1 - \frac{\mu}{\varepsilon^S})$ . As previous empirical research has demonstrated,<sup>10</sup> the elasticity of demand for housing is relatively small and always below one, with the best estimates suggesting a number around -.5. In contrast estimates of supply elasticities are typically above that number: therefore the overall impact of a reduction (increase) in the cost of capital should be negative (positive).

Yet, if the elasticity of supply  $\varepsilon^S$  is relatively small, then the land-demand effect partially compensates for the lower leasing costs of the structures. Intuitively, in order to satisfy the requirements for more housing of the population we have to push the urban boundary further, therefore increasing Ricardian rents (the value of proximity to CBD or amenities). This effect will be more pronounced in locations where a large part of the radius around the city is not developable.

Proposition 1 is of consequence for any model that uses current or past rents and the user cost formula (or similar capitalization-rate methods) to evaluate the impact of changing interest rates. In equilibrium, rental prices are directly endogenous to changes in the user cost of capital, which renders the conclusions of any such approach unreliable.

Now recall the equation characterizing housing prices *in the representative location*. Assuming no changes in the expected growth path of the city we can obtain the expected change in housing prices at  $\tilde{D}$ .<sup>11</sup>

*Proposition 2*

$$\frac{dp(\tilde{D})}{dv} \frac{v}{p(\tilde{D})} = \left( \frac{v}{v-g} \right) \left[ \sigma_p^S \cdot \frac{h(\Phi)}{h(\tilde{D})} \left( 1 - \frac{\mu}{\varepsilon^S} \right) - 1 \right] \quad (2)$$

Where  $\sigma_p^S$  is the initial *share of construction costs on housing prices in the representative loca-*

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<sup>10</sup>(Hanushek, 1980; Ermistch, Finlay, and Gibb, 1994, J.A. Barrios García, J.E. Rodríguez Hernández, 2008, Ioannides and Zabel, 2008, Fontela and Gonzalez, 2009).

<sup>11</sup>I am assuming that the expected growth path of the city boundary does not change, the assumption that all previous literature has made. In fact, lower costs of structural capital make outside locations more attractive in the short run. With high moving costs this should not imply population emigration, but will be associated with a stoppage of housing construction until the point where local productivity growth brings the city back to the old demographic path. These aspects will be modelled in future versions of the paper.

*tion.* In general, the larger the land share the bigger the positive impact of reductions in the user cost of capital. Note that if  $\sigma_p^S = 1$ -and ignoring the other effects captured by  $\frac{h(\Phi)}{h(D)} (1 - \frac{\mu}{\varepsilon^S})$ - there should be no effect of changes of interest rates on home values at all. This is due to the discounting effect of the user cost being reverse-leveraged by structural replacement costs: only land values should be affected by changes in interest rates; the leasing costs of the structures should change endogenously one-to-one with changes in user costs.

The other terms are a function of housing supply and demand elasticities. They reflect the fact that when the leasing costs of structures are reduced by lower interest rates, land rents should respond endogenously. On the one hand, the demand for land increases (according to  $\mu$ ) therefore pushing up Ricardian rents along the land supply schedule (as reflected by  $\tilde{\varepsilon}^S$ ). On the other hand, the overall structural leasing cost of housing is heavily decreasing in structure-intensive exurban areas, which makes land-intensive locations less attractive unless land rents decrease. As discussed earlier, the former effect will tend to dominate the latter in locations with inelastic land supply.

Note that the theory suggests that housing prices should increase with decreasing user costs and the effects should be positive functions of land shares. *After conditioning on land shares and housing demand elasticity*, changes in prices should also be inversely proportional to housing supply elasticities. In the next section, we will show that this was indeed the case during the housing boom period, but substantial deviations from the theory were observed. However, deviations that could not possibly be accounted for by fundamentals quickly mean-reverted during the bust.

### **3 Housing Boom and Bust: Fundamental versus Non-fundamental Components**

In this section I examine the cross-sectional housing price dynamics of the recent housing boom-bust period. I focus on a boom period that is conventionally defined as the lapse between the first quarter of 2000 and the first quarter of 2005. The bust is defined as the period between quarter 1 of 2005 and the second quarter of 2008. I stop at Q2 2008 because the economic recession, likely an aftermath of housing price fluctuations, started then (see Appendix figure 1). A global financial panic ensued the collapse of major financial institutions such as Lehman Brothers, Freddie Mac, Fannie Mae, and AIG in the 3rd quarter of 2008. It is not possible from that point to separate the intrinsic housing dynamics that may have been spurred by declines in the cost of capital (the focus of this paper) from the impact of very large economic recession and financial panic.

While considerable speculation has surrounded the justifiability of the price run-up of the early 2000s, a relatively large portion of the variability in such price growth was consistent with the theoretical model. Indeed, one of the features of the price run-up was its large heterogeneity, in itself not consistent with an alternative hypothesis *solely* based on simple and *generalized* behavioral patterns. Figures 1 and 2 show that the two key variables proposed in the model (land shares and supply elasticity) were clear predictors of relative price run ups by metropolitan area (I focus on 137 metropolitan areas for which I have both estimates of land share from Davis et al, and supply elasticities from Saiz, 2010).

Existing literature had already established that lower supply elasticities, as measured by Saiz (2010), were predictive of faster housing appreciation during the 2000-2005 boom period (Atif and Sufi and 2009,2011).<sup>12</sup> In the existing literature this is presented as consistent with a common shock to housing demand across cities that interacted with different supply conditions. However, part of this effects is actually explained by the fact that inelastic-supply locales tend also to have more expensive land in equilibrium (Saiz, 2010). Even without general equilibrium effects via new supply (i.e. no change in land rents), we should expect these land-intensive cities to experience further appreciation.

In Table 1 one I present regressions with repeat-sample FHU metropolitan housing price indexes are the dependent variable. As posited in the model, under the assumption that demand elasticities and housing size ratios are similar across locations, we can see that these two variables alone explained more than 50 percent of the variance in housing price growth. This conclusion is reinforced when looking at column two, which includes population weights. Furthermore, including past trends does not change the conclusion. In fact, lagged price growth (1970 through 2000) mean-reverted during the boom period: analysts could not have explained the relative variance in housing price inflation by arguing the continuation of previous city-specific trends of growth. This is important, insofar as it demonstrates that any city-specific story used to rationalize deviations from the model (e.g. Las Vegas may have been expected to grow much faster into the decade due to an acceleration in the growth of the tourism industry) had to be taken with a healthy dose of skepticism. Finally, and quite starkly, the two variables from the theory accounted for all regional variation in housing price growth. It is well know that, during the boom period, there was substantial variation in price growth across locales. Controlling for land share and supply elasticity renders all regional

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<sup>12</sup>Most of this literature uses estimates of supply elasticity as instruments for the price runup by metropolitan areas. This literature includes:

coefficients insignificant.

Yet, a substantial share of price growth remained unexplained by the model. I therefore separate the variance in price growth into two components: the one explained or accounted for by the model (the prediction using the coefficients in Table 1, column 1), and the unexplained portion (the residuals from the same model). In Figure 3 I plot changes in housing prices during the bust period and 2000-2005 price growth that conformed with the model; there is no discernible relationship between the two. In fact, many cities did experience further price growth. Thus, predictions from the empirical model were, on average, largely consolidated at the end of Q2 2008.

A very different picture arises when looking at the portion of the housing price boom for which the model does not account for, in Figure 4. The pattern is very stark: most of the unaccounted for growth did actually mean-revert.

I emphasize that, while mean-reversion is expected to some degree of any data series, the main result is that theoretically-accountable price growth *did not* mean-revert. Therefore, the theoretical model and empirical models that use the theory should be practical to assess the existence of *relative* local deviations from housing price fundamentals.

## 4 Conclusions

Fluctuations in housing prices are relevant to wealth accumulation, labor mobility, consumption, macroeconomic volatility, and financial market stability. However, it is *ex ante* difficult to know when housing price movements are due to fundamentals, such as changes in the user cost of capital, versus “irrational exuberance.” I propose combining the canonical urban economics Alonso-Muth-Mills model and Poterba (1984, 1990) housing asset-pricing equation to form grounded theoretical expectations about the impact of changes in the user cost of capital on home values. I show rental prices and rental expenditures to be endogenous to interest rates, which limits the applicability of conventional price-to-rent ratios. Expected changes in home values can be expressed as simple functions of the supply elasticity of housing, and the initial share of land relative to prices in a city. The simple formula can be used to diagnose and underwrite home valuations under the null hypothesis of a common shock in the user cost of capital. Empirically, I find that housing supply elasticities and land shares as of 1990 predicted 50% of the variance in price growth during the past boom. Deviations from theoretical growth mean-reverted dramatically during the bust period. However, housing price growth consistent with theoretical expectations has been largely

consolidated.

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## 5 Math Appendix

*Derivation 1*

simply consider the following productivity growth path:

$$\widetilde{w_k(\tau)} = \underline{w} + \frac{1}{(1-\mu)} \left\{ (1-\mu)t\widetilde{D} - [vcc]^{1-\mu} + \left( [vcc]^{1-\mu} + (1-\mu)t \left[ \Phi - \widetilde{D} \right] \right) e^{(1-\mu)g\tau} \right\}$$

*Proof of proposition 1*

Recall:  $H_k = \int_0^\Phi \frac{2\pi \cdot \Lambda}{h(x)} x \cdot dx = \int_0^\Phi 2\pi \Lambda r(x)^\mu \cdot x \cdot dx$ . Which implies that:  $\frac{dH_k}{v} = \int_0^\Phi 2\pi \Lambda \mu r(x)^{\mu-1} \frac{dr(x)}{dv} x dx + \frac{2\pi \cdot \Phi \Lambda}{h(\Phi)} \frac{d\Phi}{dv}$ .

Recalling that  $\frac{dr(x)}{dv} = cc \cdot \frac{h(\Phi)}{h(x)} + \frac{t}{h(x)} \frac{d\Phi}{dv}$ , and under the assumption of high moving costs (no change in population/number of households, i.e.  $\frac{\partial H_k}{\partial v} = 0$ ):

$$\begin{aligned} \frac{dH_k}{v} &= \int_0^\Phi 2\pi \Lambda \mu r(x)^{\mu-1} \left[ cc \cdot \frac{h(\Phi)}{h(x)} + \frac{t}{h(x)} \frac{d\Phi}{dv} \right] x dx + \frac{2\pi \Phi \Lambda}{h(\Phi)} \frac{d\Phi}{dv} = 0 \\ \frac{dH_k}{v} &= \left\{ \int_0^\Phi 2\pi \Lambda \mu r(x)^{\mu-1} \frac{t}{h(x)} x dx + \frac{2\pi \cdot \Phi \Lambda}{h(\Phi)} \right\} \frac{d\Phi}{dv} + \int_0^\Phi 2\pi \Lambda \mu r(x)^{\mu-1} cc \cdot \frac{h(\Phi)}{h(x)} x dx = 0 \\ \frac{dH_k}{v} &= \left\{ \int_0^\Phi 2\pi \Lambda \mu r(x)^{\mu-1} \frac{t}{h(x)} x dx + \frac{2\pi \cdot \Phi \Lambda}{h(\Phi)} \right\} \frac{d\Phi}{dv} + \int_0^\Phi 2\pi \Lambda \mu r(x)^{\mu-1} cc \cdot \frac{h(\Phi)}{h(x)} x dx = 0 \text{ which implies:} \\ \frac{dH_k}{d\Phi} \frac{d\Phi}{dv} + \mu cc h(\Phi) H_k \int_0^\Phi \frac{1}{R(x)} \left( \frac{2\pi \Lambda x}{h(x)} \frac{1}{H_k} \right) dx &= 0 \end{aligned}$$

Note that  $\frac{2\pi \Lambda x}{h(x)}$  is the number of households at each radius (area divided by average space consumption by household) and therefore that  $\int_0^\Phi \frac{1}{R(x)} \left( \frac{2\pi \Lambda x}{h(x)} \frac{1}{H_k} dx \right)$  is the average of the inverse of rental expenditures across households. Using conventional fixed-point theorems one can show that there exist a location  $\bar{D}$  such that the inverse rental expenditure equals the average inverse expenditure:  $\frac{1}{\bar{R}} \equiv \frac{1}{R(\bar{D})} = \frac{1}{r(\bar{D})h(\bar{D})} = \int_0^\Phi \frac{1}{R(x)} \left( \frac{2\pi \Lambda x}{h(x)} \frac{1}{H_k} dx \right)$ . Let us denominate  $\bar{D}$  the average-value-weighted location.

Now we can rearrange:  $\frac{d\Phi}{dv} = -\frac{\mu cc h(\Phi) \frac{1}{\bar{R}}}{\frac{dH_k}{d\Phi} \frac{1}{H_k}}$

Note that there is a one-to-one relationship between changes in the parameter  $\Phi$  and changes in rents in each location, so that we can express the following identity for changes in  $r(D)$  coming from demand shocks (increased productivity in the city) that increase the city boundary:  $\frac{dH_k}{dr(D)} = \frac{h(D)}{t} \frac{dH_k}{d\Phi}$  at any distance  $D$ .

Focusing on the average-value-weighted location we therefore obtain  $\frac{d\Phi}{dv} = -\frac{\mu cc h(\Phi) \frac{1}{\bar{R}}}{\frac{dH_k}{dr(\bar{D})} \frac{1}{H_k} \frac{1}{r(\bar{D})h(\bar{D})} \frac{t}{r(\bar{D})h(\bar{D})}} =$

$-\frac{\mu c h(\Phi)}{\frac{dH_k}{dr(\tilde{D})} \frac{r(\tilde{D})}{H_k} t}$ . We can now define the supply-elasticity of housing as  $\varepsilon^S \equiv \frac{dH_k}{dr(\tilde{D})} \frac{r(\tilde{D})}{H_k}$ , which corresponds to the percentage change in the number of homes that arises from a 1 percent change in the average-value-weighted rents induced by an exogenous change in city productivity (demand for the city). Introducing back to the expression that focuses on changes in rents:

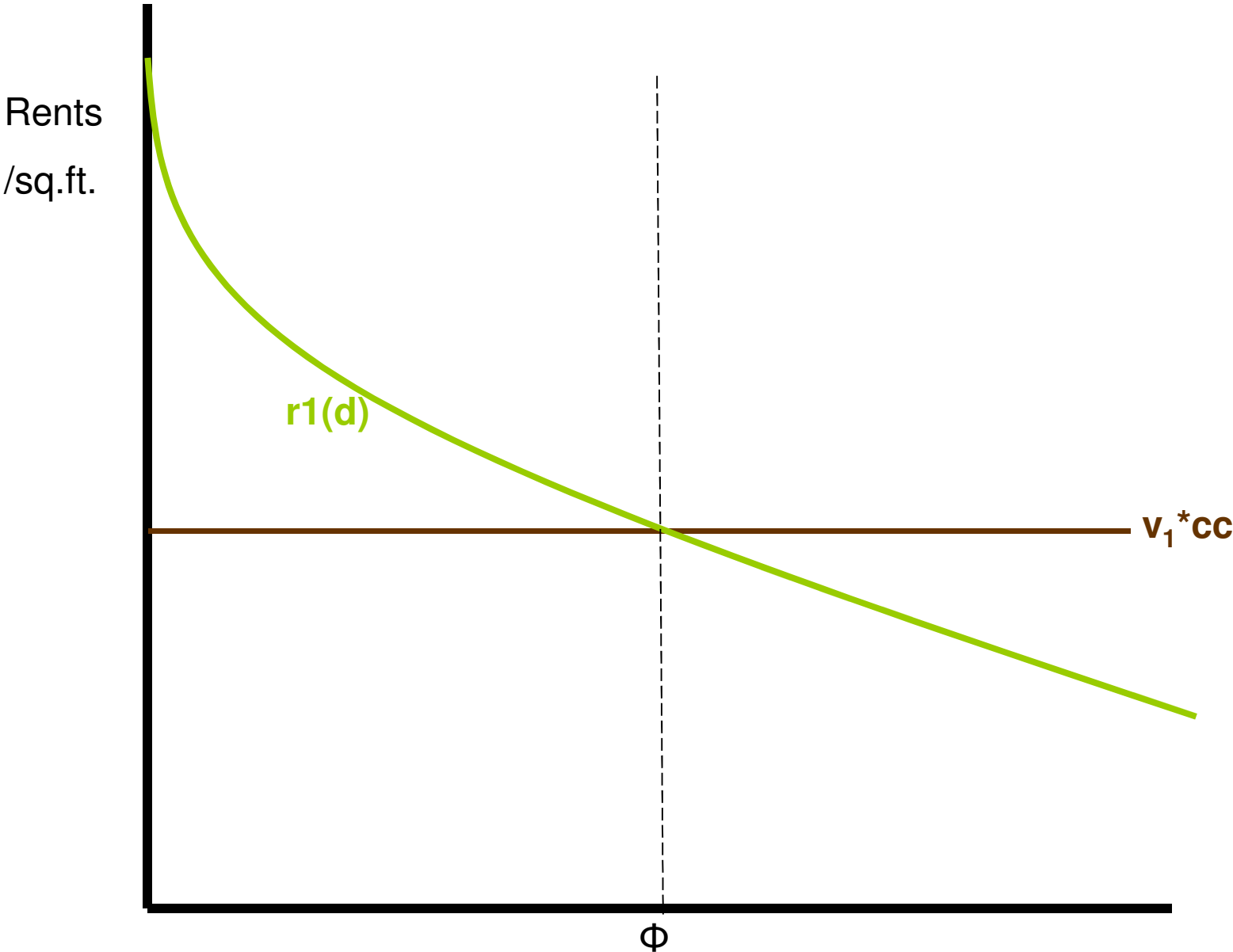
$$\frac{dr(D)}{dv} = cc \cdot \frac{h(\Phi)}{h(D)} - cc \frac{h(\Phi)}{h(D)} \frac{\mu}{\varepsilon^S} = cc \frac{h(\Phi)}{h(D)} \left(1 - \frac{\mu}{\varepsilon^S}\right), \text{ and so } \frac{dr(D)}{dv} \frac{v}{r(D)} = \frac{vcc}{r(D)} \frac{h(\Phi)}{h(D)} \left(1 - \frac{\mu}{\varepsilon^S}\right)$$

*Proof of proposition 2*

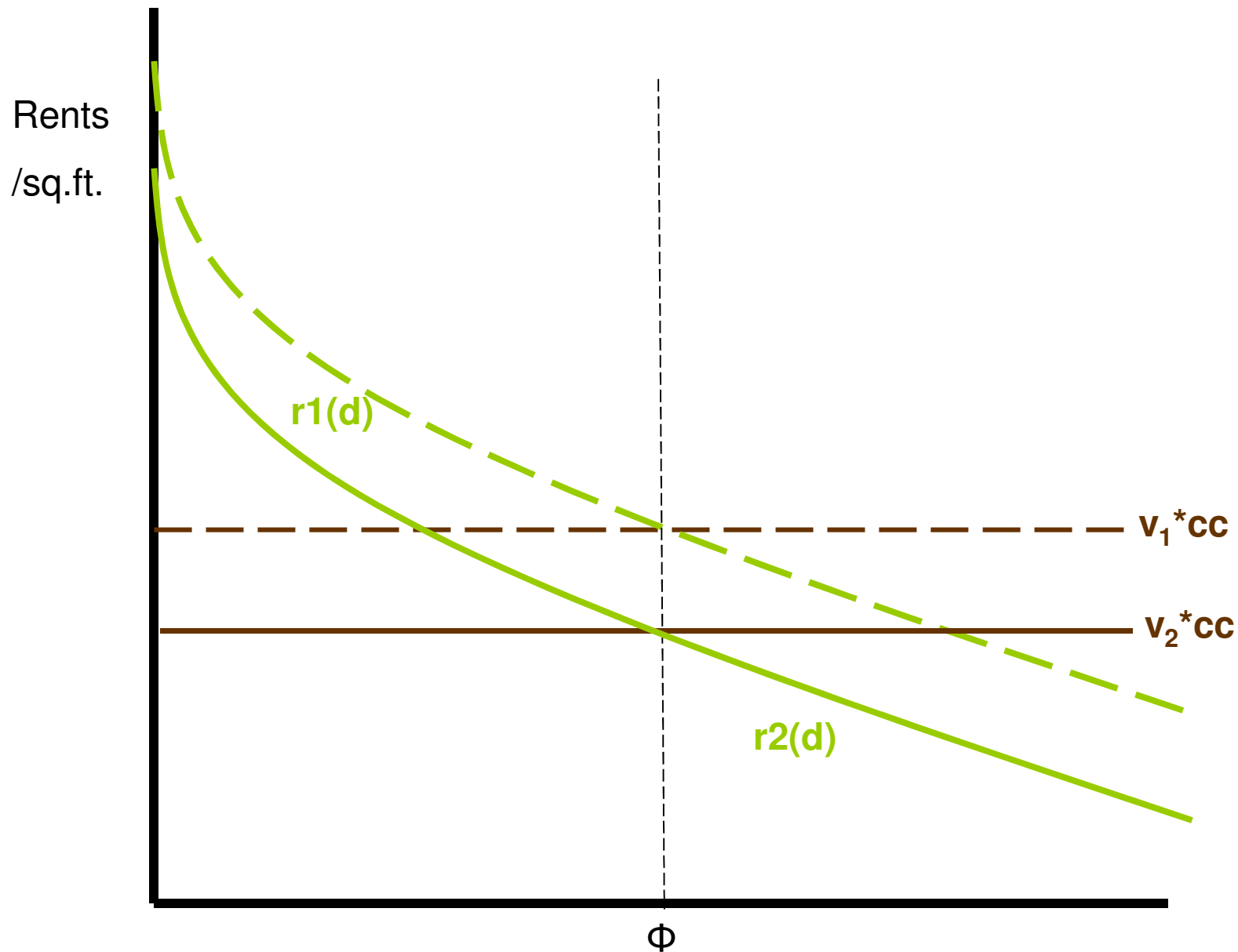
$$\text{Recall that } p(\tilde{D}) = \frac{r(\tilde{D})}{v-g}, \text{ and therefore } \frac{dp(\tilde{D})}{dv} = -\frac{r(\tilde{D})}{(v-g)^2} + \frac{1}{v-g} \frac{dr(\tilde{D})}{dv} = -\frac{1}{v-g} p(\tilde{D}) + \frac{1}{v-g} \left[ cc \frac{h(\Phi)}{h(D)} \left(1 - \frac{\mu}{\varepsilon^S}\right) \right]$$

$$\frac{dp(\tilde{D})}{dv} = \frac{1}{v-g} \left[ cc \frac{h(\Phi)}{h(D)} \left(1 - \frac{\mu}{\varepsilon^S}\right) - p(\tilde{D}) \right], \text{ and so } \frac{dp(\tilde{D})}{dv} \frac{v}{p(\tilde{D})} = \frac{v}{v-g} \left[ \frac{cc}{p(\tilde{D})} \frac{h(\Phi)}{h(D)} \left(1 - \frac{\mu}{\varepsilon^S}\right) - 1 \right]$$

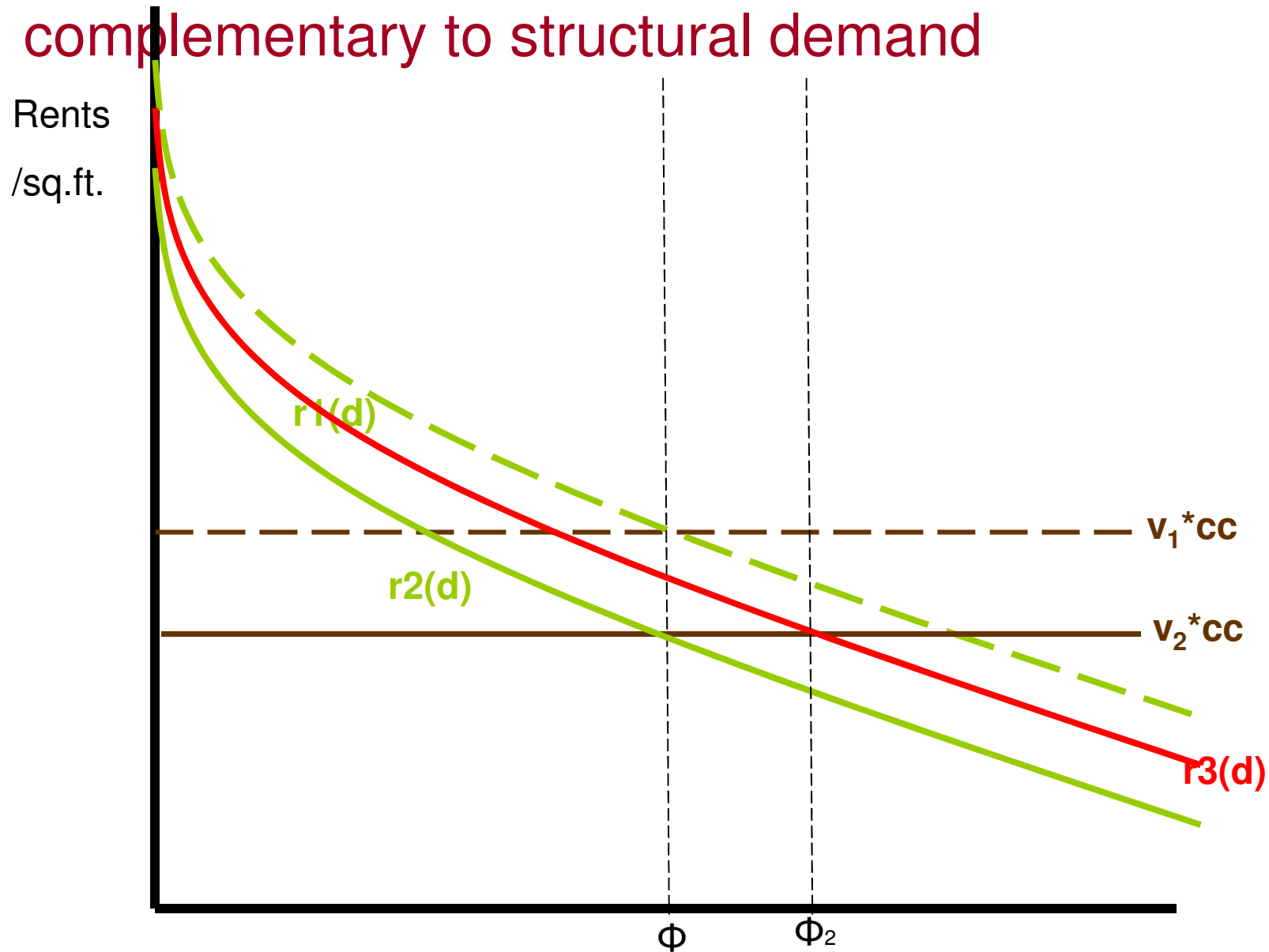
# Graph A: rents and prices per square foot



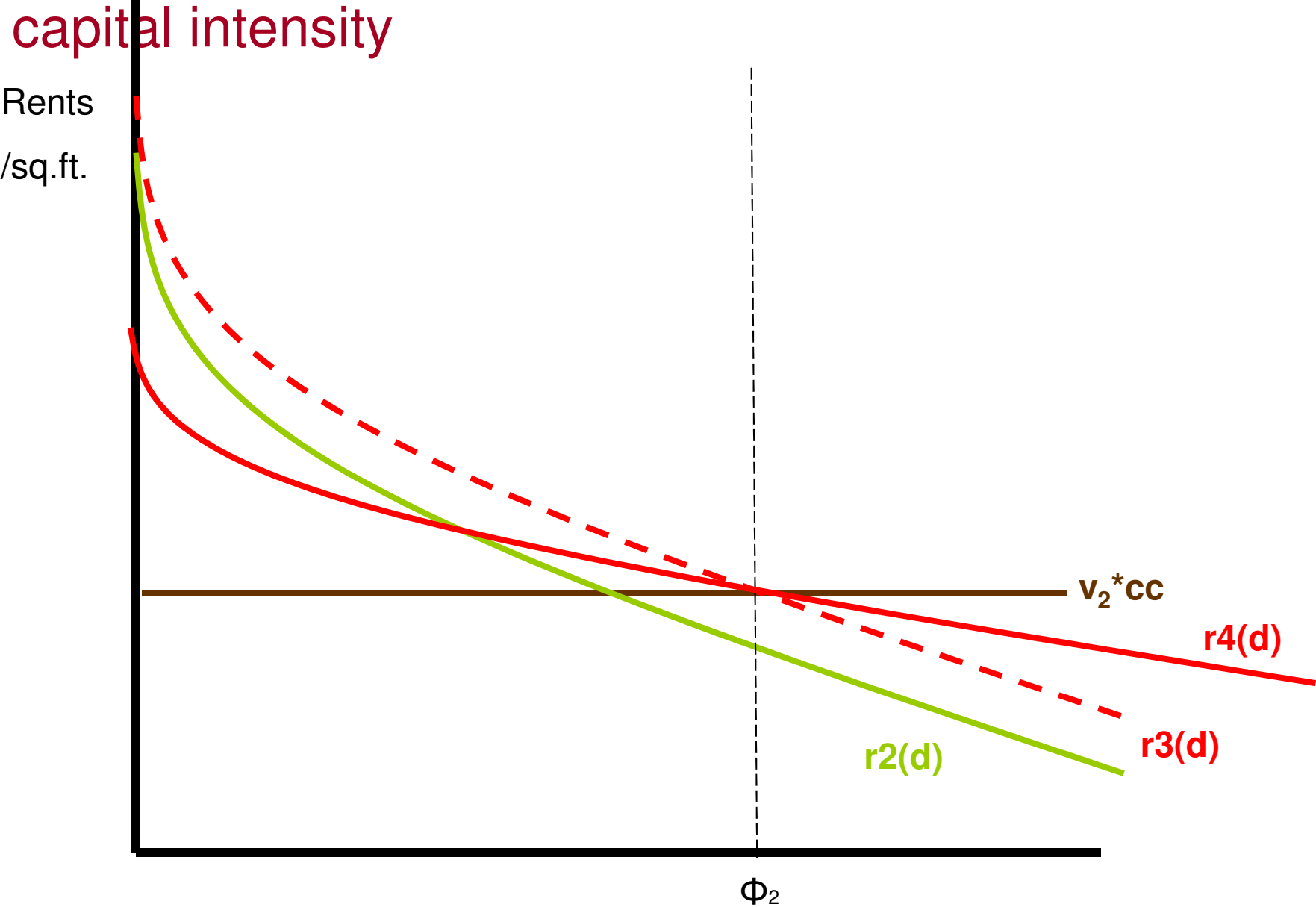
# Graph B: rents and prices per square foot: arbitrage



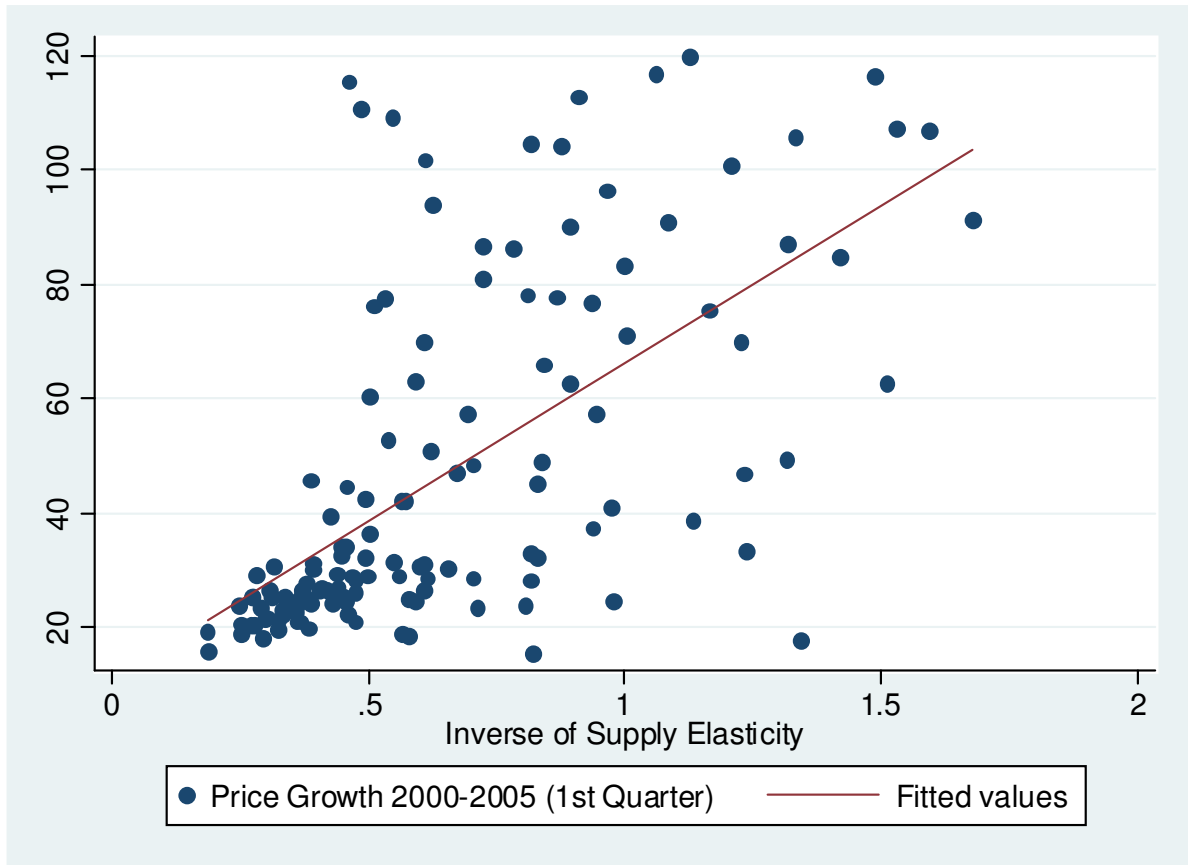
# Graph C: rents and prices per square foot: land is complementary to structural demand



# Graph D: compensate central locations for low capital intensity

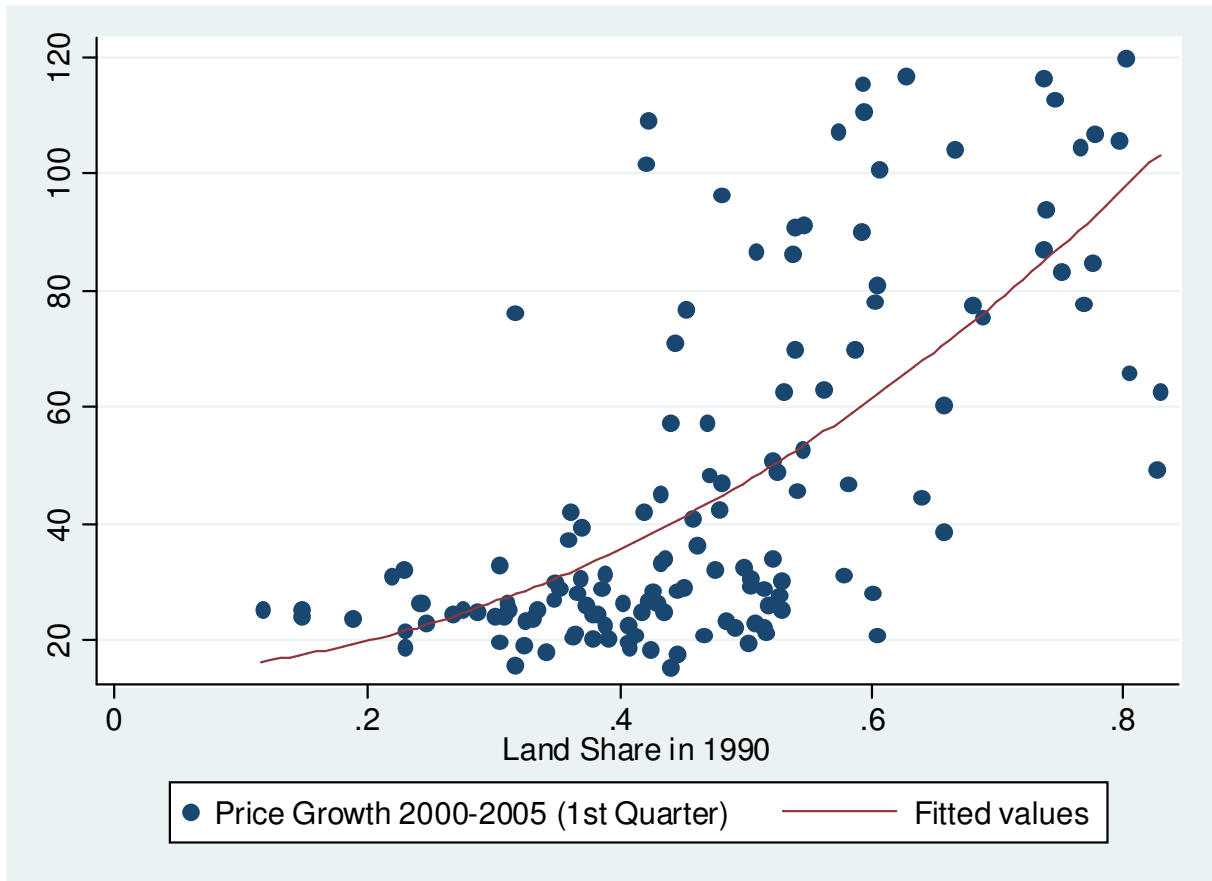


**Figure 1: Price Growth During the Boom and Supply Elasticity**

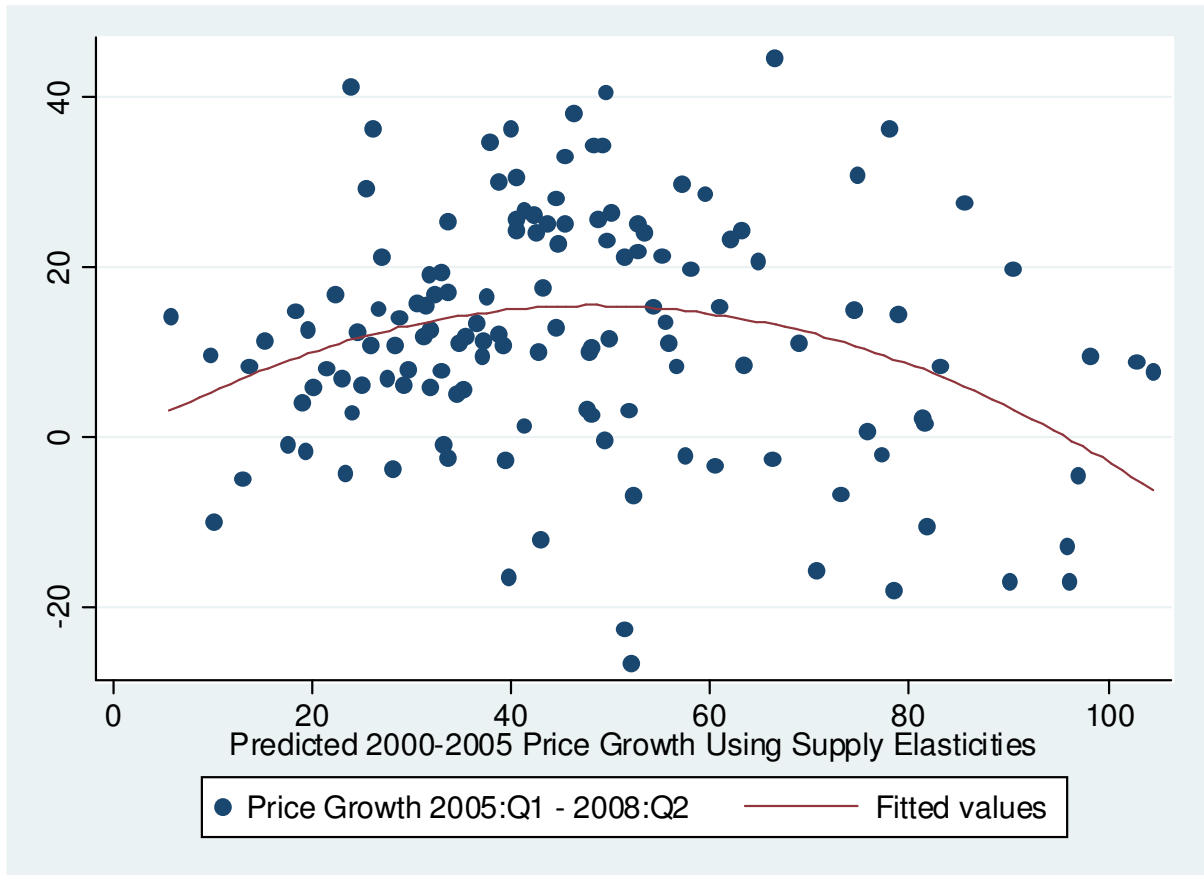




**Figure 2: Price Growth During the Boom and land shares in 1990**



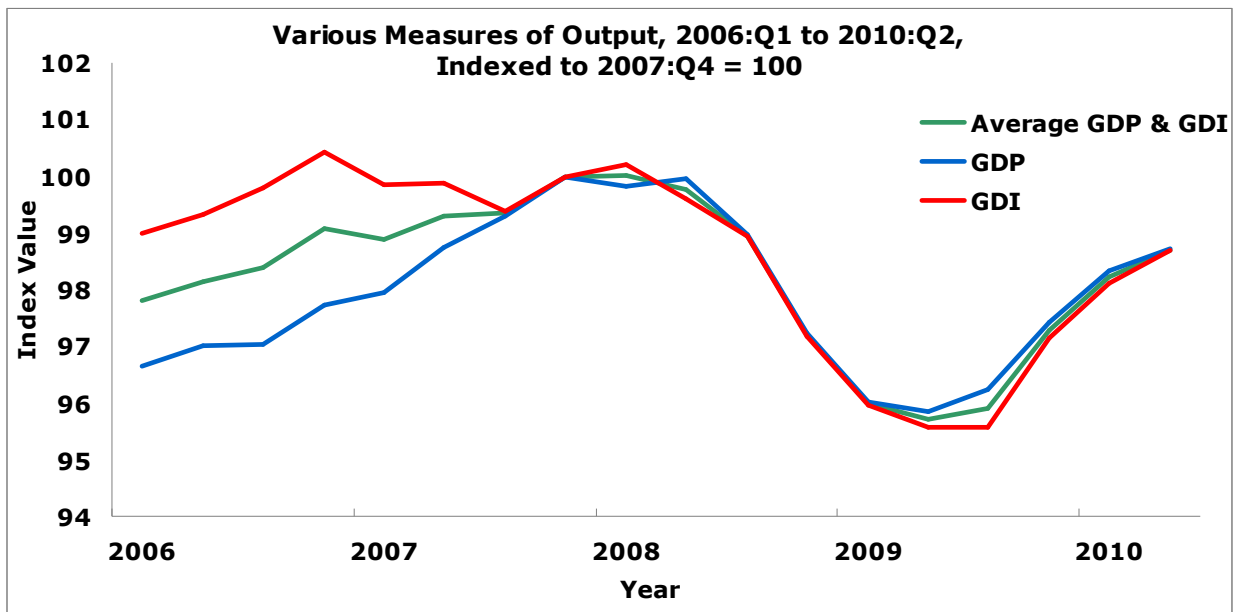
**Figure 3: Expected Growth using Fundamentals and Bust**



**Figure 4: Growth Unaccounted for by Fundamentals and Bust**



**Appendix Figure 1**



Source: NBER

**TABLE 1***Real Estate Boom and Fundamentals*

	Price Growth 2000-2005 (1st Quarter)			
	(1)	(2)	(3)	(4)
Inverse of Supply Elasticity	31.445 (6.605) <sup>***</sup>	26.013 (5.549) <sup>***</sup>	32.138 (6.556) <sup>***</sup>	22.349 (6.418) <sup>***</sup>
Land Share in 1990	85.802 (14.574) <sup>***</sup>	99.675 (14.654) <sup>***</sup>	104.469 (17.572) <sup>***</sup>	65.224 (15.150) <sup>***</sup>
Log Price 2000 - Log Price 1970			-18.631 (9.993) <sup>*</sup>	
Middle Atlantic				-1.535 (20.037)
East North Central				-10.304 (20.207)
West North Central				-5.673 (20.979)
South Atlantic				2.53 (19.732)
East South Central				-17.299 (20.722)
West South Central				-17.963 (20.631)
Mountain				-14.237 (20.117)
Pacific				14.931 (19.594)
Specification	OLS	Pop weights	Past Trends	Regional effects
Observations	137	137	137	137
R-squared	0.52	0.6	0.53	0.62

Standard errors in parentheses

\* significant at 10%; \*\* significant at 5%; \*\*\* significant at 1%