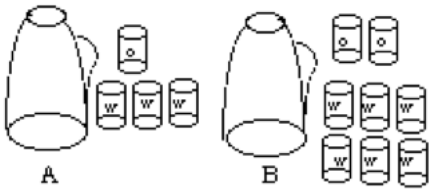
Math **Oranginess** Day 1

You should first work on each problem individually and then discuss your answer with your team. Treat these problems as thinking exercises, rather than relying solely on computational procedures. Be ready to explain your reasoning to the whole group.

(1) Suppose there are two pitchers with orange drink at the lunch table. The orange drink in pitcher A is made by mixing 1 can of orange concentrate with 3 cans of water. The mixture in pitcher B is made by mixing 2 cans of orange concentrate with 6 cans of water. Which will taste more *orangey*, the mixture in pitcher A or the mixture in pitcher B?

(2) In a fifth-grade class some students reasoned about the "Orangey Juice" situation (as pictured) in the following ways:



i. A student said that the mixture in pitcher A is more orangey because less water went into making it.

ii. Other students in the same class argued that the mixture in pitcher A has 2 more cans of water than orange concentrate, and pitcher B has 4 more cans of water than concentrate. Therefore, pitcher A is more orangey because it has more orange concentrate.

iii. There was another student who argued as follows: Pour 1 can of orange concentrate and 1 can of water into pitcher A. Take 2 cans of orange concentrate and 2 cans of water and pour them into pitcher B. The two mixtures are equally orangey because they are made with equal parts of orange and water, that is, 50-50. Now there are only 2 cans of water left to go into pitcher A and 4 cans of water left to go into pitcher B. Because the mixtures are the same strength, when you add 4 cans of water to the mixture in pitcher B, it will be more watery than the mixture in pitcher A, which gets only 2 more cans of water. Therefore the mixture in pitcher A is more orangey.

(a) Analyze each case making sure you understand the student's thinking. Determine the type of reasoning the student is engaged in.

(b) For each case, can you change the problem to help the student see where their approach fails.

(c) How can *oranginess* be quantified in this situation to facilitate the comparison?

(3) Consider the following **new situation**:

a. Suppose now that one can of water and one can of orange concentrate is added to pitcher B, as in part (1). Will the mixtures compare in the same way as before? Why? Explain your reasoning.

b. A third pitcher, pitcher C, is made by mixing equal parts of pitchers A and B from part (1). Which one will taste more orangey: pitcher A, pitcher B, or pitcher C?

(4) Suppose now that we do not know how pitchers A and B were made exactly, but pitcher C is made again from equal parts of pitchers A and B.

a. What can you say about the relative "oranginess" between A and B, if pitcher C is more orangey than pitcher A?

b. What can you say if pitcher C is more orangey than pitcher B?