Abstractionism / Neologicism

April 26–27, 2014

UConn Logic Group
University of Connecticut, Storrs — logic.uconn.edu/workshop/

Saturday, April 26

9:00 – 10:00	Sean Walsh (UC Irvine): *The Constructible Universe, the Naive Conception, and Intensional Logic*
10:10 – 11:10	Kevin C. Klement (UMass–Amherst) *A Generic Russellian Elimination of Abstract Objects*
11:20 – 12:20	Shay Logan (Minnesota) *Categories for the Neologicist*
12:20 – 2:00	lunch (roaming into "town")
2:00 – 3:00	Stewart Shapiro (OSU) *Frege meets Aristotle: points as abstracts*
3:10 – 4:10	Roy T. Cook (Minnesota) *Abstraction Principles and Four Notions of Invariance*
4:20 – 5:20	Øystein Linnebo (Oslo, N.I.P. Aberdeen) *Recarving of content*
5:30 – 7:00	**Keynote:** Crispin Wright (N.I.P. Aberdeen, NYU) *Abstraction and Epistemic Entitlement: On the Epistemological Status of Hume's Principle*
7:45	conference dinner

Sunday, April 27

9:00 – 10:00	Graham Leach-Krouse (Kansas State) *Structural Abstraction Principles*
10:10 – 11:10	Fernando Ferreira (Lisboa) *Impredicativity and Fregean arithmetic*
11:20 – 12:20	Francesca Boccuni (Milano) *A Theory of Fregean Concepts and Objects*
12:30 – 1:30	Richard G. Heck, Jr (Brown): *Is Frege's Definition of the Ancestral Correct?*
1:30 – 2:30	lunch (provided) and *Roundtable Discussion*

Abstracts

Francesca Boccuni (Università Vita Salute San Raffaele, Milano)
A Theory of Fregean Concepts and Objects

I will present a stratified plural theory for Fregean objects based on Boolos's intuition that Fregean abstraction principles rest on explicit existential assumptions. The interaction of the axioms of the theory is so tweaked that the resulting system is consistent, but strong enough to deliver some important Fregean abstraction principles (e.g. BLV and HP) and PA2 via Frege's definitions. In order to motivate the restrictions imposed on the axioms for the sake of consistency, the intensional nature of concepts and their relation with pluralities is investigated, to the effect that a novel motivation for stratification of concepts is suggested.

Roy T. Cook (Minnesota)
Abstraction Principles and Four Notions of Invariance

One particularly interesting type of abstraction principle are those principles where the equivalence relation providing identity conditions for the abstracts governed by the abstraction principle is purely logical. Isolating and studying such purely logical abstraction principles requires identifying exactly which equivalence relations are 'logical' in the relevant sense. In this paper I compare and contrast four potential accounts of 'logicality', and show that the more strict the notion of logicality we adopt, the more central cardinal numbers and counting becomes to the theory of abstracts that results.

Fernando Ferreira (Universidade de Lisboa)
Impredicativity and Fregean arithmetic

In Frege's logicism, numbers are logical objects in the sense that they are extensions of certain concepts. Even though Frege's logical system is inconsistent, it was shown by Richard Heck that its restriction to predicative (second-order) quantification is consistent. This predicative fragment is, nevertheless, too weak to develop arithmetic. In this talk, I will consider an extension of Heck's system with impredicative quantifiers. In this extended system, both predicative and impredicative quantifiers co-exist but it is only permissible to take extensions of concepts formulated in the predicative fragment of the language. This system is consistent. Moreover, it proves a form of reducibility applied to concepts true of only finitely many objects. With the aid of this form of reducibility, it is possible to develop arithmetic in a Fregean way.

Richard G. Heck, Jr (Brown)

Is Frege's Definition of the Ancestral Correct?

Why should one think that Frege's definition of the ancestral is correct? It can be proven to be extensionally correct, but the argument uses arithmetical induction, and that fact might seem to undermine Frege's claim to have justified induction in purely logical terms—a worry that goes back to Bruno Kerry and Henri Poincaré. In this paper, I discuss such circularity objections and then offer a new definition of the ancestral, one that is intended to be intensionally correct; its extensional correctness then follows without proof. It can then be proven to be equivalent to Frege's definition, without any use of arithmetical induction. This constitutes a proof that Frege's definition is extensionally correct that does not make any use of arithmetical induction, thus answering the circularity objections.

Graham Leach-Krouse (Kansas State)

Structural Abstraction Principles

In this paper, I will present a class of "structural" abstraction principles suggested by certain formal features of the approach to abstraction favored by Dedekind and Cantor. I'll then go on to show how these principles can be put to use in the general theory of abstraction, by proving some illustrative basic results about the class of structural abstraction principles. In particular, I'll show how a certain theorem of Shelah gives us a useful sufficient condition for consistency in the structural setting, and how this consistency condition suffices to solve what Neo-Logicists call the "bad company" problem in a large class of important cases. In addition, I'll show how, in the structural setting, we can get a good measure of the logical strength of our abstraction principles by making use of categories (in the mathematical sense) of interpretations between theories.

Øystein Linnebo (Oslo, N.I.P. Aberdeen)

Recarving of content

The idea of content recarving has played a central role in Fregean and neo-Fregean logicism. I first articulate some logical constraints on content recarving and illustrate these by means of some purported examples of content recarving. Then I examine how to make philosophical sense of the idea of content recarving and argue that none of the existing proposals succeeds.

Shay Logan (Minnesota)

Categories for the Neologicist

An abstraction principle that serves to define categories is introduced. It is seen to be inconsistent in the expected ways. Various attempts to make it consistent are explored.

Kevin C. Klement (UMass–Amherst)
A Generic Russellian Elimination of Abstract Objects

Recently, there's been interest in "abstractionist" forms of logicism which attempt to deduce mathematical theories from abstraction principles, those of the form:

$$f(x) = f(y) \text{ iff } Rxy$$

where R is an equivalence relation, and f is a functor mapping entities x and y in the same logical type to the same abstract object just in case R holds between them. In this paper I explore (but do not definitely endorse) a position on which it is possible to eliminate the need for principles postulating abstract objects by treating the terms in such an abstraction principles as "incomplete symbols", using Russell's no-classes theory as a template from which to generalize. I defend views of this stripe against certain objections, most notably Richard Heck's charge that syntactic forms of nominalism cannot correctly deal with non-first-orderizable quantification over apparent abstracta.

Stewart Shapiro (The Ohio State University)
Frege meets Aristotle: points as abstracts

There has been a fair amount of work developing a "gunky" or point-free (or point-less) account of space, or space-time. Geoffrey Hellman and I are working on one such account. In such frameworks, it is common to define "points" as the limits of infinite sequences of regions, following Whitehead's "extensive abstraction". I will show how to characterize points, within the Hellman-Shapiro gunky background, as neo-logicist abstractions from regions. This helps make sense of some Aristotelian conceptions of continuity and his talk of points as the endpoints of intervals.

Sean Walsh (Department of Logic and Philosophy of Science, UC Irvine)
The Constructible Universe, the Naive Conception, and Intensional Logic

This paper studies the relationship between three foundational systems: Gödel's Constructible Universe of Sets, the naive conception of set found in consistent fragments of Frege's Grundgesetze, and the intensional logic of Church's Logic of Sense and Denotation. One basic result shows how to use the constructible sets to build models of fragments of Frege's Grundgesetze from which one can recover these very constructible sets using Frege's definition of membership. This result also allows us to solve the related consistency problem and joint consistency problems for abstraction principles with limited amounts of comprehension. In addition, we are able to prove the theory consisting of all these abstraction principles with limited amounts of comprehension interprets a fragment of set theory when coupled with a form of global choice. Another basic aim of this paper is to show how to "factor" these results via a consistent fragment of Church's Logic of Sense and Denotation: so one may use the constructible sets to build models of Church's Logic of Sense and Denotation, from which one may then define models of the consistent fragments of Frege's Grundgesetze.

Crispin Wright (N.I.P. Aberdeen, NYU)
Abstraction and Epistemic Entitlement:
On the Epistemological Status of Hume's Principle

The abstractionist programme of foundations for classical mathematical theories is, like its traditional logicist ancestors, first and foremost an epistemological project. Its official aim is to demonstrate the possibility of a certain uniform mode of a priori knowledge of the basic laws of arithmetic, real and complex analysis, and set theory. Traditional logicism aimed to show that mathematical knowledge could be accomplished using only analytic definitions and theses of pure logic and hence is not merely *a priori* if logic is but is effectively a proper part of logic. Abstractionism, however, adds abstraction principles to the base materials employed in the traditional logicist project—principles that, at least in the central, interesting cases, are neither pure analytic definitions nor theses of pure logic as conventionally understood. Thus, whatever significance they may carry for the prospects for logicism, the epistemological significance of technically successful abstractionist projects must turn on the epistemological status of the abstraction principles used, with any demonstration of a priority in particular being dependent on whether those principles can themselves rank as knowable a priori. My primary focus here will be on this natural thought.