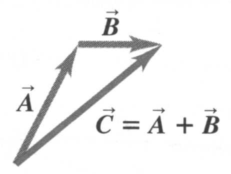
The Physics Learning Resource Center Reference sheet

Vectors

- The sum, $\boldsymbol{A}+\boldsymbol{B}$, of two vectors, $\boldsymbol{A}$ and $\boldsymbol{B}$, is the vector created by placing the tail of $\boldsymbol{B}$ on the head of $\boldsymbol{A}$ and then drawing a line segment from $\boldsymbol{A}$'s tail to $\boldsymbol{B}$'s head:



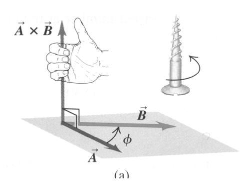
- To determine the difference between two vectors, rewrite the equation as a sum: $\boldsymbol{B}=\boldsymbol{C}-\boldsymbol{A}$ can be rewritten as $\boldsymbol{A}+\boldsymbol{B}=\boldsymbol{C}$, so $\boldsymbol{B}$ in the above illustration is the difference between $\boldsymbol{C}$ and $\boldsymbol{A}$.
- $\boldsymbol{A}$ and $\boldsymbol{B}$ are *components* of $\boldsymbol{C}$ because they add up to $\boldsymbol{C}$. Components that are perpendicular to each other are particularly useful. We commonly use $\boldsymbol{i}$, which points in the positive x-direction, $\boldsymbol{j}$, which points in the positive y-direction, and $\boldsymbol{k}$, which points in the positive z-direction. All three vectors have a length of one.
- The dot product, $\boldsymbol{A}\cdot\boldsymbol{B}$, between two vectors, $\boldsymbol{A}$ and $\boldsymbol{B}$, is the real number

$$\boldsymbol{A}\cdot\boldsymbol{B}=AB\cos\theta=A_xB_x+A_yB_y+A_zB_z.$$

  Some useful dot products are

$$\boldsymbol{i}\cdot\boldsymbol{i}=\boldsymbol{j}\cdot\boldsymbol{j}=\boldsymbol{k}\cdot\boldsymbol{k}=1.$$

  All other possible dot products between $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are zero.
- The cross product, $\boldsymbol{A}\times\boldsymbol{B}$, between two vectors, $\boldsymbol{A}$ and $\boldsymbol{B}$, is a vector whose length is $\boldsymbol{A}\times\boldsymbol{B}=AB\sin\theta$.
  You find the direction of the cross product using the right-hand rule: point the fingers of your right hand along the direction of $\boldsymbol{A}$ and hold your hand so that you can turn it towards $\boldsymbol{B}$. Your thumb is pointing in the direction of the cross product.



- The vector area of a surface is a vector whose length is equal to the area of the surface and that points "out" at ninety degrees from the surface.
- **A CALCULUS CONCEPT** The path integral of a vector function of position, $\boldsymbol{f}(\boldsymbol{r})$, along a path, $\mathcal{P}$, defined by the curve $(x(\lambda), y(\lambda), z(\lambda))$, is

$$\int_{\mathcal{P}}\boldsymbol{f}(\boldsymbol{r})\cdot d\boldsymbol{r}=\int\left(f_x\frac{dx}{d\lambda}+f_y\frac{dy}{d\lambda}+f_z\frac{dz}{d\lambda}\right)d\lambda.$$

Some Basic Considerations

- Draw a picture to help you visualize the problem. Label it with the algebraic variables for the quantities given to you.
- Think of what relationships exist between what you are given and what is needed.
- Work the problem with the algebraic variables for as long as possible. Only insert numbers at the end.
- Use units when inserting the numbers and make sure they match correctly.
- Use significant digits correctly while doing calculations.

Fundamental Kinematic Quantities

- t is the instant of time that we are looking at the system.
- t_0 is the instant that the initial conditions of the system were set; often has a value of zero.
- $\boldsymbol{r}(t)\equiv x(t)\boldsymbol{i}+y(t)\boldsymbol{j}+z(t)\boldsymbol{k}\equiv r(t)\hat{\boldsymbol{r}}(t)$ is the position vector. It indicates the particle's location and it is time-dependent.
- $\boldsymbol{r}_0\equiv\boldsymbol{r}(t_0)$ is the initial position of the particle.
- $\Delta\boldsymbol{r}=\boldsymbol{r}-\boldsymbol{r}_0$ is the displacement vector.
- **A CALCULUS QUANTITY** $d\boldsymbol{r}$ is the infinitesimal displacement.
- **A CALCULUS QUANTITY**

$$\boldsymbol{v}=\frac{d\boldsymbol{r}}{dt}$$

  is the instantaneous velocity of the particle. Its magnitude, v, is the instantaneous speed of the particle. $\boldsymbol{v}_0\equiv\boldsymbol{v}(t_0)$ is the initial velocity of the particle.
- $$\boldsymbol{v}_{\text{ave}}=\frac{\Delta\boldsymbol{r}}{t-t_0}\equiv\frac{\Delta\boldsymbol{r}}{\Delta t}$$

  is the average velocity of the particle over the period of time Δt. Its magnitude, v_{ave}, is the average speed of the particle.
- **A CALCULUS QUANTITY**

$$\boldsymbol{a}=\frac{d\boldsymbol{v}}{dt}$$

  is the instantaneous acceleration of the particle.
- $$\boldsymbol{a}_{\text{ave}}=\frac{\Delta\boldsymbol{v}}{\Delta t}$$

  is the average acceleration of the particle over the the period of time Δt.

The Basic Kinematic Equations

For a particle experiencing constant acceleration

$$\boldsymbol{r}(t)=\tfrac{1}{2}\boldsymbol{a}t^2+\boldsymbol{v}_0t+\boldsymbol{r}_0.$$
$$v^2=v_0^2+2\boldsymbol{a}\cdot\Delta\boldsymbol{r}.$$
$$\boldsymbol{v}=\boldsymbol{a}t+\boldsymbol{v}_0.$$
$$\boldsymbol{v}_{\text{ave}}=\frac{\boldsymbol{v}+\boldsymbol{v}_0}{2}.$$

Fundamental Dynamic Quantities

- m is the mass of the particle; *both* its tendency to resist acceleration and its interaction strength with gravitational fields.
- q is the charge of the particle; its interaction strength with electric fields.

Newton's Laws

1. If there is no net force on a particle, it experiences no acceleration. The particle is in *equilibrium.*
2. Let $\sum\boldsymbol{F}$ be the net force on the particle—the vector sum of all the forces acting on it. This net force produces an acceleration on the particle determined by $\sum\boldsymbol{F}=m\boldsymbol{a}$.
3. If Particle 1 exerts a force on Particle 2, Particle 2 exerts a force equal in magnitude and opposite in direction on Particle 1.

Contact Forces

- When a particle exerts a force on a surface, the surface feels the component of that force that is perpendicular, or *normal*, to the surface. By Newton's third law, the surface exerts a force, $\boldsymbol{N}$, equal and opposite to this normal component.
- The frictional force, $\boldsymbol{f}$, has maximum magnitude μN.
- The fluid resistance, $\boldsymbol{f}$ has a velocity-dependent magnitude, so the basic kinematic equations do not apply.

$$f=\begin{cases}kv, & \text{low velocities,}\\ Dv^2. & \text{high velocities.}\end{cases}$$

- The buoyant force, $\boldsymbol{B}$, is upward with a magnitude equal to the weight of the fluid displaced.
- A spring "stretched" a distance, x, (a negative value means the spring is squished) exerts a force, $\boldsymbol{F}=-kx\boldsymbol{i}$.

Forces Mediated Through Fields

- The force exerted on a particle with mass, m, from a gravitational field, $\boldsymbol{g}(\boldsymbol{r})$, at a position, $\boldsymbol{r}$, is $\boldsymbol{F}=m\boldsymbol{g}(\boldsymbol{r})$.
- The force exerted on a particle with charge, q, with a velocity, $\boldsymbol{v}$, from an electric field, $\boldsymbol{E}(\boldsymbol{r})$, and a magnetic field, $\boldsymbol{B}(\boldsymbol{r})$, at a position, $\boldsymbol{r}$, is

$$\boldsymbol{F}=q\left[\boldsymbol{E}(\boldsymbol{r})+\boldsymbol{v}\times\boldsymbol{B}(\boldsymbol{r})\right].$$

  This is the Lorentz force equation.

Classical Fields

- The gravitational field, $\boldsymbol{g}(\boldsymbol{r})$, at a position, $\boldsymbol{r}$, created by a point source mass, M, at the origin is

$$\boldsymbol{g}(\boldsymbol{r})=-G\frac{M}{r^2}\hat{\boldsymbol{r}}.$$

  A USEFUL APPROXIMATION At an altitude of less than 1100 meters, $\boldsymbol{g}$ is downward with a constant value of $g=9.81\ \mathrm{m/s^2}$.
- The electric field, $\boldsymbol{E}(\boldsymbol{r})$, at a position, $\boldsymbol{r}$, created by a point source charge, Q, at the origin is

$$\boldsymbol{E}(\boldsymbol{r})=\frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{\boldsymbol{r}}.$$

- The magnetic field, $\boldsymbol{B}(\boldsymbol{r})$, at a position, $\boldsymbol{r}$, created by a point source charge, Q, at the origin and moving with a velocity, $\boldsymbol{v}$, is

$$\boldsymbol{B}(\boldsymbol{r})=\frac{\mu_0}{4\pi}\frac{Q\boldsymbol{v}\times\hat{\boldsymbol{r}}}{r^2}.$$

Angular Kinematics and Dynamics

Everything that happens in a straight line has its equivalent when rotating:

Linear Quantity	*Angular Quantity*
t	t
x	θ
v	ω
a	α
m	I
q	q
F	τ

The linear and angular quantities are connected by the relationship $s=r\theta$ when the rotational motion is circular.

Work Energy Theorem

$$W\equiv\int_{\mathcal{P}}\boldsymbol{F}(\boldsymbol{r})\cdot d\boldsymbol{r}=\Delta K\equiv\tfrac{1}{2}m_fv_f^2-\tfrac{1}{2}m_iv_i^2.$$

Potential Energy

If $\boldsymbol{F}$ is a conservative force and $\mathcal{P}$ is any path from an arbitrary location, called *ground*, to the point determined by $\boldsymbol{r}$, then the potential energy, $U(\boldsymbol{r})$, from $\boldsymbol{F}$ is

$$U(\boldsymbol{r})=-\int_{\mathcal{P}}\boldsymbol{F}(\boldsymbol{r})\cdot d\boldsymbol{r}=-W_{\text{Ground to }\boldsymbol{r}}.$$

The potential energy at ground is always zero.

- With ground infinitely far away,
  - The potential energy from gravity is

$$U(r)=-G\frac{mM}{r}.$$

    A USEFUL APPROXIMATION At an altitude of less than 1100 meters, the potential energy at a height, h, is

$$U(h)=mgh.$$

  - The potential energy from an electric field is

$$U(r)=\frac{1}{4\pi\epsilon_0}\frac{qQ}{r}.$$

- The potential energy from a magnetic field is zero because *magnetic forces never do work.*
- The potential energy of a spring is

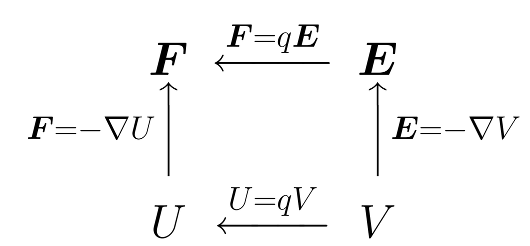
$$U(x)=\frac{1}{2}kx^2.$$

Potential

When potential energy comes from a field, there exists a quantity known as *potential* (not to be confused with potential energy). The electric potential, $V(r)$, is

$$V(r)=\frac{1}{4\pi\epsilon_0}\frac{Q}{r}.$$

Note that, of $\boldsymbol{F}$, $\boldsymbol{E}$, U, and V, the potential is the easiest quantity to calculate. The four quantities are related by



Gauss's Law

$$\Phi\equiv\oint_{\mathcal{G}}\boldsymbol{E}(\boldsymbol{r})\cdot d\boldsymbol{a}=\frac{Q_{\text{inside}}}{\epsilon_0}.$$

THE TRICK In situations where the charge distribution reduces the spatial dependency of $\boldsymbol{E}$, draw a Gaussian surface that keeps $\boldsymbol{E}$ constant and pull it outside the integral.